Beyond the Van der Waals EoS of hadronic matter

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(with **V. Sagun**, **K. Bugaev**, **E. Nikonov**, **A. Taranenko** and **G. M. Zinovjev**)

arXiv:1704.06846 [nucl-th]
Nucl. Phys. A 924, 24 (2014)

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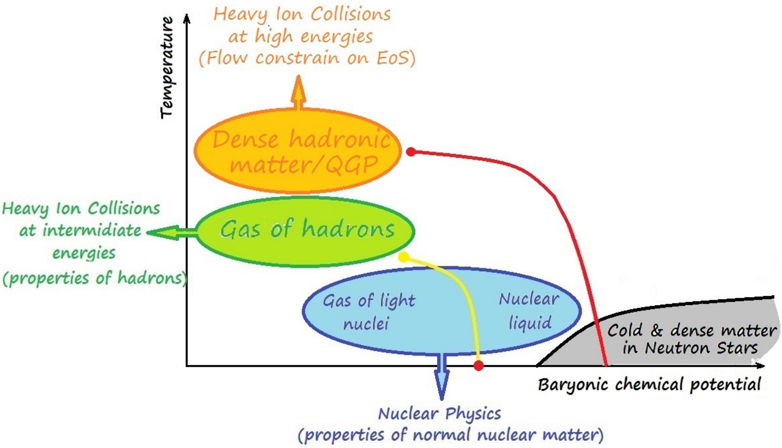
Outline

1. Motivation

Outline

Outline

Strongly interacting matter phase diagram



- **Constrains on EoS come from several regions of phase diagram**
- **Wide applicability range of EoS is required**
- **Practical need to go beyond the standard approximations**

Constrains on hadronic EoS

- Many particle species $\Rightarrow$ Grand Canonical Ensemble
- Properties of symmetric nuclear matter at ground state

$$T = 0,\ \ n_0 = 0.16\ fm^{-3} \Rightarrow \frac{E_{binding}}{A} = 16\ MeV,\ \ p = 0$$

- Causality up to the densities where QGP is expected

$$c_s = \sqrt{\frac{dp}{d\epsilon}} \leq 1 \ \ \Rightarrow \text{EoS is not too stiff}$$

- Flow constrain on the strongly interacting matter EoS

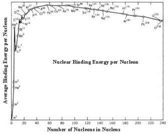
  P. Danielewicz, R. Lacey, W. G. Lynch,, Science 298, 1593 (2002)

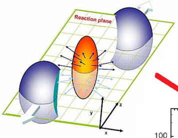
- Hard core radii of hadrons
  - nucleon-nucleon scattering data $\Rightarrow R \simeq 0.3$ fm

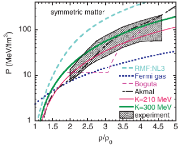
    A. Bohr, B. Mottelson, Nucl. Structure (Benjamin, NY 1969), V.1, p. 266.
  - hadron production in A+A collisions $\Rightarrow R \lessapprox 0.5$ fm

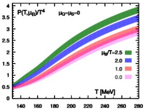
    A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A 772, 167 (2006)
    V. V. Sagun, Ukr. J. Phys. 59, 755 (2014)
    V. Vovchenko, H. Stoecker, J.Phys. Conf. Ser. 779 (2017)
- Low temperature thermodynamics of lattice QCD

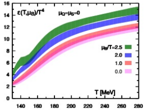
  A. Bazavov et al., Phys. Rev. D 95, 054504 (2017)









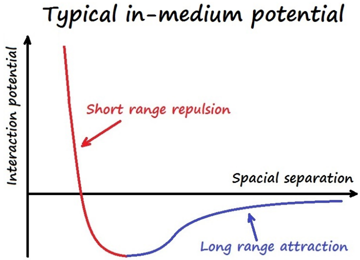


Ingredients of phenomenological EoS

- **Realistic interaction**

short range repulsion:
1. finite size of particles
2. exchange interaction via repulsive channels (not significant at $T > 50\ MeV$)

long range attraction:
1. exchange interaction via attractive channels (suppressed at $T > 50\ MeV$)



3. Finite size of particles $\Rightarrow$ hard core repulsion

4. Exchange interaction $\Rightarrow$ 1. microscopic $\mathcal{L}$ in mean-field approximation
   2. thermodynamically consistent potentials

- **Quantum statistics** (obvious but not always trivial)
- **Many particle species**

Hadronic hard core

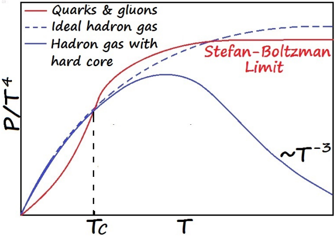
- Prevents phenomenological EoS of QCD from quark confinement at high temperatures

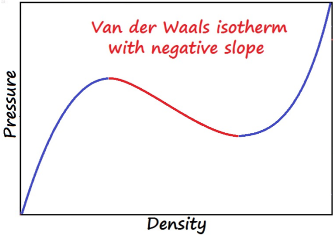
$$ideal\ gas: \quad p \sim T^4$$

$$hard\ core: \quad p \sim T$$

- Accounts for short range repulsion between the constituents (hadrons, nuclear fragments, etc.)

- Is necessary for statistical (not Van der Waals) liquid-gas phase transition in cluster models





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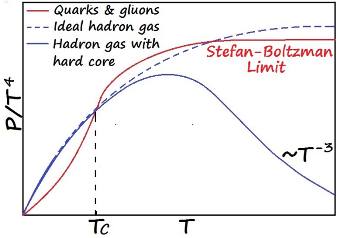

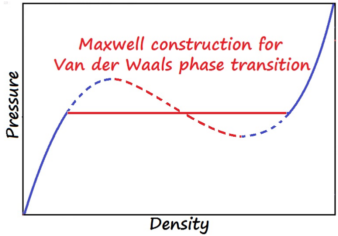

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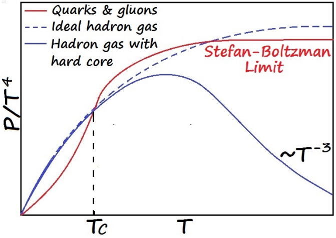
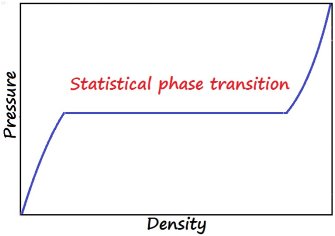




Hadronic hard core repulsion in heavy ion collisions

- **Important element in description of particle yields**

  Ideal hadron gas is proven to be inadequate at high A+A collision energies

  J. Cleymans and H. Satz, Z. Phys. C 57, 135 (1993)

  J. Cleymans, M.I.Gorenstein, J. Stalnacke and E.Suhonen, Phys. Scripta 48, 277 (1993)

- **Models with exact analytical solution in finite volume** (not in this talk)

  K. A. Bugaev, Acta. Phys. Polon. B 36, 3083 (2005)

  K. A. Bugaev, A. I. Ivanytskyi, V. V. Sagun and D. R. Oliinychenko, PEPAN 10, 6, 508 (2013)

  - analogs of phase transition and critical point in finite volume
  - charge fluctuations in finite volume

EoS with constant excluded volume

- EoS with constant excluded volume b

$$p = \sum_{particles} p_{id}(\mu) \quad \Rightarrow \quad p = \sum_{particles} p_{id}(\mu - pb)$$

D.H. Rischke, M.I. Gorenstein, H.Stoecker and W. Greiner, Z. Phys. C 51(1991) 485

- Example: onecomponent Van der Waals EoS ($b = 4v_0, \; v_0$ - volume of hard sphere)

$$V \to V - V_{excl} \Rightarrow \overbrace{p = nT = \frac{NT}{V}}^{ideal\ gas} \to \frac{NT}{V - V_{excl}} = \frac{NT}{V - Nb} = \frac{nT}{1 - nb}$$

$$n = \frac{\partial p}{\partial \mu} = \frac{p}{T + pb} \quad \Rightarrow \quad p = \underbrace{T \int \frac{d\vec{k}}{(2\pi)^3} \exp\left(\frac{\mu - pb - \sqrt{m^2 + k^2}}{T}\right)}_{p_{id}(\mu - pb)}$$

Whether excluded volume is constant?

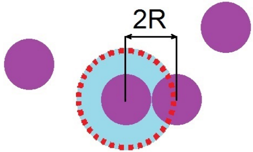
Excluded volume

- Excluded volume per particle

$$v_{excl} \equiv \frac{V_{excl}}{N}, \quad where\ N\ is\ number\ of\ particles$$

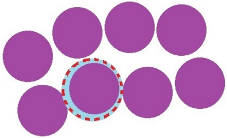
- Translation of one particle around another

Low densities

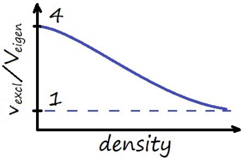


$$v_{excl} \simeq \frac{1}{2} \cdot \frac{4\pi}{3}(2R)^3 = 4v_0$$

High densities



$$v_{excl} \simeq v_0$$



What is dependence of $\mathbf{v_{excl}}$ on density?

How $\mathbf{v_{excl}}$ enters EoS?

Virial expansion of EoS with hard core repulsion

- Low density expansion of one component EoS (valid, when $n \simeq n_{ideal\ gas}$)

$$\frac{p}{T} = n + a_2 n^2 + a_3 n^3 + a_4 n^3 + \ldots$$

- Virial coefficients a_k are defined by the potential of microscopic interaction

R.K.Pathria, Statistical Mechanics, Pergamon Press, Oxford, 1972

Boltzmann ideal gas: $a_2 = 0,\ a_3 = 0,\ a_4 = 0,\ \ldots$

Boltzmann hard spheres: $a_2 = 4v_0,\ a_3 = 10v_0^2,\ a_4 = 18.365v_0^3,\ \ldots$

Have to be reproduced within realistic phenomenological EoS

..., but it is not trivial if:

- effects of quantum statistics are accounted
- many particle species are included

- Carnahan-Starling EoS for hard spheres

$$\frac{p}{nT} = \frac{1 + nv_0 + (nv_0)^2 - (nv_0)^3}{(1 - nv_0)^3}$$

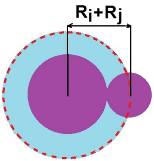
- 7 first virial coefficients are reproduced
- recent generalization to quantum case
  D. Anchishkin and V. Vovchenko, J. Phys. G 42, 105102 (2015)
- NO generalization to many particle species

Mixture of many Boltzmann particle species

$$\frac{p}{T} = n + a_2 n^2 + a_3 n^3 + a_4 n^3 + \ldots$$

- Excluded volume for different particle species at low density

$v^{ij}_{excl} = \frac{1}{2} \cdot \frac{4\pi}{3}(R_i + R_j)^3$ – matrix of second virial coefficients



- Quantitites of the Boltzmann ideal gas

$$p = nT, \quad n = \sum_i n_i^{id}, \quad n_i^{id} = g_i \int \frac{d\vec{k}}{(2\pi)^3} \exp\left(\frac{\mu_i - \sqrt{m_i^2 + k^2}}{T}\right)$$

- Virial expansion for many particle species with $v_i = \frac{4\pi}{3} R_i^3$ and $s_i = 4\pi R_i^2$

$$\frac{p}{T} = \overbrace{\sum_i n^{id}}^{\simeq n} - \overbrace{\sum_{i,j} v^{ij}_{excl} n_i^{id} n_j^{id}}^{\simeq a_2 n^2} + \ldots = \sum_i n_i^{id} \left(1 - \overbrace{v_i \sum_j n_j^{id}}^{bulk\ \ term} - \overbrace{s_i \sum_j n_j^{id} R_j}^{surface\ \ term}\right) + \ldots$$

Boltzmann EoS with induced surface tension

$$\frac{p}{T} = \sum_i n_i^{id} \left(1 - \overbrace{v_i \sum_j n_j^{id}}^{bulk\ \ term} - \overbrace{s_i \sum_j n_j^{id} R_j}^{surface\ \ term}\right) + \ldots$$

- Bulk term – first order of **pressure** $\frac{p}{T} = \sum\limits_i n_i^{id} + \mathcal{O}(n^2)$
- Surface term – first order of **surface tension** $\frac{\Sigma}{T} = \sum\limits_i n_i^{id} R_i + \mathcal{O}(n^2)$
- High density extrapolation (gives exponentials)

$$\begin{cases} \frac{p}{T} = \sum\limits_i n_i^{id} \left(1 - \frac{p v_i}{T} - \frac{\Sigma s_i}{T}\right) \\ \frac{\Sigma}{T} = \sum\limits_i n_i^{id} \left(1 - \frac{p v_i}{T} - \frac{\alpha \Sigma s_i}{T}\right) R_i \end{cases} \Rightarrow \begin{cases} \frac{p}{T} = \sum\limits_i n_i^{id} \exp\left(-\frac{p v_i + \Sigma s_i}{T}\right) \\ \frac{\Sigma}{T} = \sum\limits_i n_i^{id} \exp\left(-\frac{p v_i + \alpha \Sigma s_i}{T}\right) R_i \end{cases}$$

$\alpha > 1$ accounts for not uniqueness of extrapolation to high densities

V.Sagun, A.Ivanytskyi, K. Bugaev, I. Mishustin, Nucl. Phys. A 924, 24 (2014)

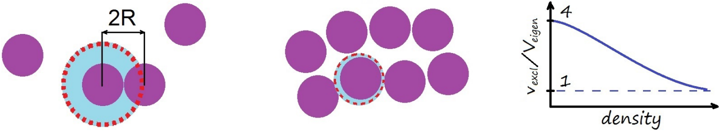
Excluded volume in one component Boltzmann EoS

- Equation of State

$$\begin{cases} \frac{p}{T} = n^{id} \exp\left(-\frac{pv_0+\Sigma s_0}{T}\right) \\ \frac{\Sigma}{T} = n^{id} \exp\left(-\frac{pv_0+\alpha\Sigma s_0}{T}\right) R \end{cases} \Rightarrow \frac{p}{T} = n^{id} \exp\left(-\frac{\overbrace{pv_0\left(1+3e^{\frac{(1-\alpha)\Sigma s}{T}}\right)}^{v_{excl}}}{T}\right)$$

- Excluded volume per particle

$$v_{excl} = v_0\left(1+3e^{\frac{(1-\alpha)\Sigma s}{T}}\right) \rightarrow \begin{cases} 4v_0, & \Sigma \rightarrow 0 \\ v_0, & \Sigma \rightarrow \infty \end{cases}$$



$\alpha > 1$ switches excluded volume regimes

Virial expansion of one component Boltzmann EoS

- Virial expansion of one component EoS with induced surface tension

$$
\begin{aligned}
p &= nT\Big[1 + \overbrace{4v_0}^{a_2} n + \overbrace{\Big(16 - 18(\alpha-1)\Big)v_0^2}^{a_3} n^2 \\
&+ \underbrace{\Big(64 - 216(\alpha-1) + \frac{243}{2}(\alpha-1)^2\Big)v_0^3}_{a_4} n^3\Big] + \mathcal{O}(n^5)
\end{aligned}
$$

- Second virial coefficient of hard spheres $a_2 = 4v_0$ is reproduced always

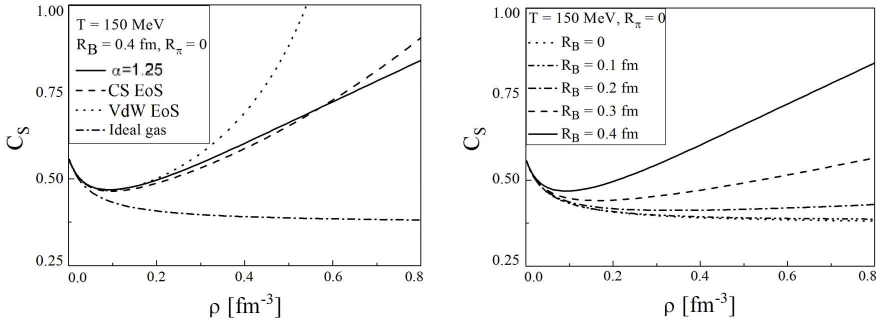
- Third virial coefficient of hard spheres $a_3 = 10v_0^2 \Rightarrow \alpha = \frac{4}{3}$
  a_4 – not reproduced

- Fourth virial coefficient of hard spheres $a_4 \simeq 18.365 v_0^3 \Rightarrow \alpha \simeq 1.245$
  a_3 – reproduced with 16% accuracy

One parameter reproduces two (3rd and 4th) virial coefficients and allows generalization to multicomponent case

Causality of IST EoS at very extreme cases

- Mixture of baryons (N and Δ) and pions



- The widest causality interval (up to 7 normal nuclear densities) at $\alpha = 1.25$

Virial expansion and causality analysis give the same value of α

Mean filed potential

- External mean filed $\Rightarrow$ medium dependent contribution U to one particle energy

$$\sqrt{m^2+k^2} \to \sqrt{m^2+k^2}+U$$

- Contribution of particle of sort i to p:

$$p_i^{id}\left(\mu_i - pv_i - \Sigma s_i\right) \to p_i^{id}\left(\mu_i - pv_i - \Sigma s_i - U(n_i^{id})\right) + p_{int}(n_i^{id}) \textit{ in expression for } p$$

$$p_i^{id}\left(\mu_i - pv_i - \alpha\Sigma s_i\right) \to p_i^{id}\left(\mu_i - pv_i - \alpha\Sigma s_i - U(n_i^{id})\right) + p_{int}(n_i^{id}) \textit{ in expression for } \Sigma$$

- Thermodynamic consistency (p and Σ can not depend on μ and n simultaneously)

$$n\frac{\partial U(n)}{\partial n} = \frac{\partial p_{int}(n)}{\partial n} \Rightarrow \text{thermodynamic identity } n = \frac{\partial p}{\partial \mu} \text{ is respected}$$

K. A. Bugaev and M. I. Gorenstein, Z. Phys. C 43, 261 (1989)

EoS with Induced Surface Tension

$$\begin{cases} p = \sum_i \left[p_i^{id}\left(T, \mu_i - U(n_i^{id}) - pv_i - \Sigma s_i\right) + p_{int}(n_i^{id}) \right] \\ \Sigma = \sum_i \left[p_i^{id}\left(T, \mu_i - U(n_i^{id}) - pv_i - \alpha\Sigma s_i\right) + p_{int}(n_i^{id}) \right] R_i \end{cases}$$

- **Many sorts of particles**
- **Quantum statistics** is accounted by construction of p^{id} and n^{id}
- **Realistic interaction**: hard core repulsion between particles
  mean-field potential U
- **Virial coefficients** are reproduced
- **Causal behavior** up to densities where QGP is expected

Generalized Walecka model for nuclear matter

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\sigma + \mathcal{L}_\omega + \overline{N}(g_\sigma \sigma - g_\omega \not{\omega})N$$

- Additional mean filed attraction $U(n) = -C_d^2 \sqrt{n}$
- Hard core repulsion is accounted within the IST framework
- EoS in mean field approximation (point like mesons)

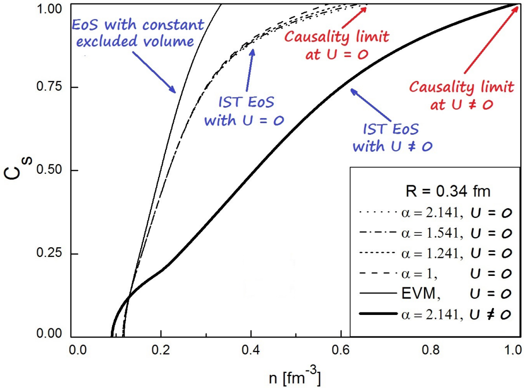
$$p = p^{id}\left(\mu^* - U(n^{id}) - pv_0 - \Sigma s_0\right) + p_{int}(n^{id}) - \frac{m_\sigma^2 \sigma^2}{2} + \frac{m_\omega^2 \omega^2}{2}$$

$$\Sigma = \left[p_{id}\left(\mu^* - U(n^{id}) - pv_0 - \alpha \Sigma s_0\right) + p_{int}(n^{id})\right] R$$

- **Parameters**: g_σ, g_ω and C_d – properties of normal nuclear matter
  α – widest range of causality (next slide)
  R - flow constrain on nuclear matter EoS

Causality of Walecka model at $T = 0$

- Quantum virial coefficients of hard spheres are not known $\Rightarrow$ α has to be fixed from the widest causality range condition

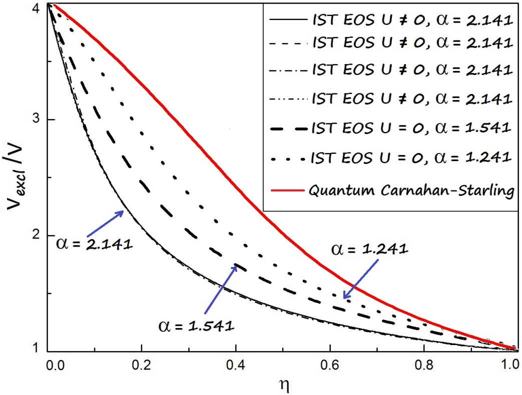


$\alpha = 2.141$ provides widest range of causality in quantum case

($n_{lim} = 3.5 n_0$ at $U = 0$ and $n_{lim} = 6.5 n_0$)

Excluded volume per particle at $T = 0$

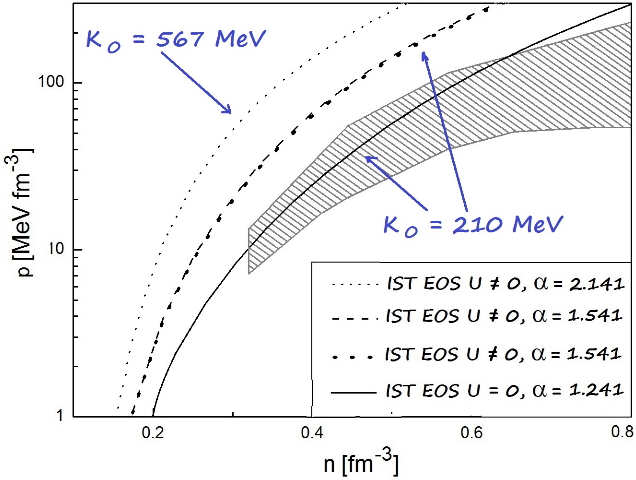
- Packing fraction $\eta = n \cdot \frac{4}{3}\pi R^3$



$\alpha = 2.141$ corresponds to softest EoS (even without attraction)

v_{excl} is defined only by α

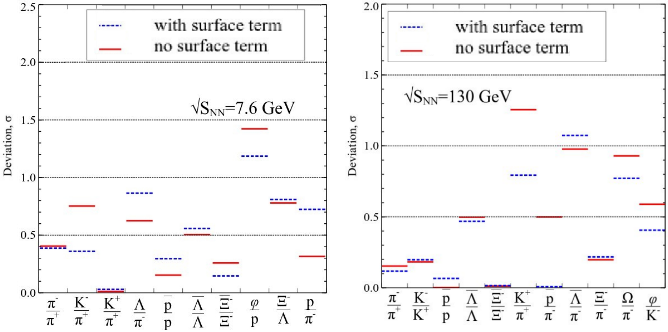
EoS at $T = 0$



Flow constrain is fulfilled at $K_0 = 210$ MeV

Hadron Resonance Gas

- Hadrons with masses ≤ 2.5 GeV (widths, strong decays, zero strangeness)
- Hard-core repulsion is accounted within the IST framework, no explicit attraction
- 111 independent particle ratios measured at 14 energies from 2.7 to 200 GeV/A
- 14×4 local parameters $(T, \mu_B, \mu_{I3}, \gamma_s)$ + 5 global parameters (hard core radii)

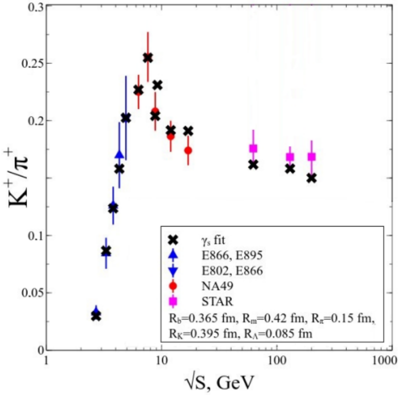


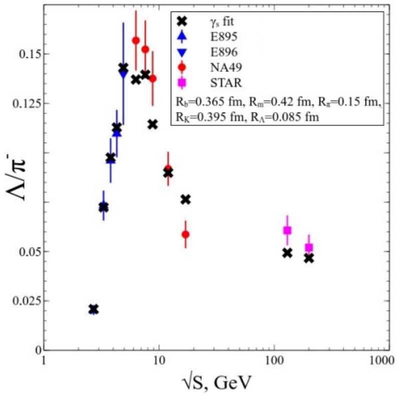
$$R_b = 0.365\ fm,\ R_m = 0.42\ fm,\ R_\pi = 0.15\ fm,\ R_K = 0.395\ fm,\ R_\Lambda = 0.085\ fm$$

$$Overall\ \chi^2/dof \simeq 1.038$$

K^+/π^+ and Λ/π^- ratios

- Even the most problematic ratios are under description now



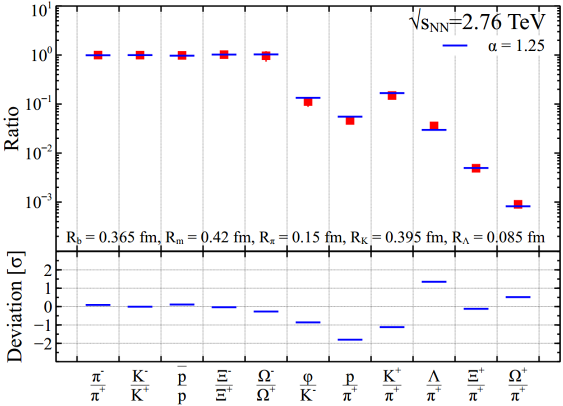


$$\frac{\chi^2_{K^+/\pi^+}}{dof} = \frac{3.29}{14}$$

$$\frac{\chi^2_{\Lambda/\pi^-}}{dof} = \frac{11.62}{12}$$

Hadron Resonance Gas at ALICE energies

- 11 independent particle yields
- 1 parameter temperature (5 hard core radii were fixed before)
- Overal $\chi^2/dof \simeq 1.038$
- Freeze out temperature $T_{FO} = 154 \pm 7\ MeV$



Conclusions

- Multicomponent EoS in Grand Canonical Ensemble
- Virial coefficients of Boltzmann hard spheres
- Quantum Statistics
- Ground state of normal nuclear matter
- High quality description of particle yields measured in A+A collisions
- Wide range of causality
- Flow constrain on nuclear matter equation of state

Thank you for attention